Mathematics Standards Committee

Second Addendum to November 4, 2004 Report to the New York State Commissioner of Education

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ADDENDUM #2 TO NOVEMBER 4, 2004 REPORT

I. INTRODUCTION

The Mathematics Standards Committee issued its first report in November 2004. That report recommended that the draft be sent to the field for comment. There was a month-long comment period from November 4, 2004 to December 4, 2004. Over 2,000 comments were received, many representing entire departments or faculties. The Committee reviewed all of the comments and revised the performance indicators where it felt this was appropriate. The enormity of responding to over 2,000 comments led the Committee to the decision that it would be wise to divide the work. Because the Prekindergarten through Grade 8 performance indicators will be implemented next school year (2005-2006) in preparation for the NYS Testing Program Grades 3-8, we focused first on those grade levels. In January 2005, the Committee forwarded its first addendum in which it recommended adoption of the Prekindergarten through Grade 8 performance indicators, as revised after review of the field comments. The Prekindergarten through Grade 8 performance indicators were adopted by the Board of Regents on January 5, 2005.

This report is the second and final addendum to the original report. It contains the Committee's recommendations for high school performance indicators revised after reviewing field comment[1], alignment modifications to the Prekindergarten through Grade 8 performance indicators, and recommendations for graduation requirements.

Although the work was divided out of necessity, the Committee envisions its report and the two addenda as one entity, and wishes to emphasize the importance of, and its continuing support for, the recommendations made in the earlier reports, notably those concerning technology, curriculum development and staff development.

II. RECOMMENDATIONS

A. The Three High School Courses

This work started with the Independent Math A Panel. The field input to that Panel indicated a lack of understanding about "Math A" and "Math B," which were designed as examinations, not courses. Schools and districts attempted to develop courses designed to prepare students for these state examinations, but the titles and course content differed from district to district. The field input also indicated that high school

[1] For a lengthy review of the field comment and how the Committee responded to this feedback, please refer to our first Addendum, dated January 4, 2004

mathematics programs should be conceptualized as three one-year courses of mathematics for the typical student, not as three years of coursework punctuated by an examination at the halfway point (after a year and a half of instruction). The purpose of the Committee's first recommendation is to respond to that feedback. If approved, it will result in high school mathematics programs in New York State consisting of three one-year courses, with the content for each course clearly identified by the title.

Recommendation 1. The Committee recommends that high school mathematics programs in New York State include three one-year courses of instruction, with the course titles to be, "Algebra," "Geometry," and "Algebra 2 and Trigonometry."

Since these course titles are commonly understood in the field of mathematics, there would be a common understanding of high school mathematics programs in New York State both statewide and nationally through acceptance of this recommendation.

B. The Performance Indicators for the Three Courses

The Mathematics Standards Committee has researched mathematics programs from across the nation, and from around the world, as it has drafted performance indicators for these three courses. The first draft of the performance indicators received many comments from the field, which has led to revisions. Some of the issues addressed include the following.

First, from the feedback, it is clear that the original draft did not communicate clearly enough our view of the importance of conceptual understanding in mathematics. Our initial thinking was that the process performance indicators would be brought in as part of the curriculum development work, which will follow the Regents adoption of the new learning standard in mathematics. However, when the field responded that the content performance indicators without the process performance indicators focused the reader exclusively on procedural knowledge, we decided it was important to add process performance indicators to clarify the Committee's intent. While this addendum is designed to be read in conjunction with our earlier reports, one point stated in our first addendum is so important that we feel the need to repeat it here:

The study of mathematics is a study of ideas and concepts. Yes, students need to know the procedures, but the knowledge of those procedures without conceptual understanding is surface knowledge that is virtually meaningless.

Therefore, the Committee added process performance indicators to the three high school courses.

Second, the field has responded positively to the concept of three courses, each with a focus on a specific area of mathematics. There was concern expressed in the feedback that the courses not drift over time into courses like those with similar (or the same) names taught decades ago, courses in which memorization and rote learning were the norm. The latest research in mathematics instruction makes it clear that we must go far beyond the procedural mathematics taught to many of us years ago. By including process performance indicators, we believe we have clarified that our intent is to recommend a problem-solving approach, which we believe is essential to a deep understanding of mathematics. By focusing each course on a specific area of mathematics, there will be time to explore the concepts in more depth.

Third, a related concern in the comments on the November draft of the geometry performance indicators is that respondents felt we were recommending a return to the memorization of "statement-reason" geometric proofs. This feedback was extraordinarily valuable, as such a notion was the last thing in our minds, and showed to us a serious miscommunication in that first draft. Developing a correct logical trail from a hypothesis to a conclusion is an essential learning in mathematics. Students must learn how to develop such logical trails, and also must also be able to distinguish between correct logical arguments and faulty logical arguments. Students should know that, unlike a court of law in which "proof" means "beyond a reasonable doubt," in mathematics, if something is proven based on other known facts using correct logic, the conclusion is not simply "proved beyond a reasonable doubt," it becomes proven fact. While we believe students need to understand the essence of mathematical reasoning and proof, and that they need to be able to apply this knowledge to situations which are new to them, we do not believe they must formally prove every geometric relationship. For many students, such a course would be dull and boring, and certainly would not accomplish the goals of this Committee. We envision students:

- exploring geometric relationships,
- discussing with classmates what relationships can be deduced from the knowledge given,
- working with physical models of plane figures and solids, and
- using available software in their explorations.

The Committee has rewritten many of the geometry performance indicators to communicate clearly what it believes should be the essence of this course.

During its review of the high school performance indicators, the Committee noted that minor adjustments to the Regents-approved Prekindergarten through Grade 8 performance indicators would result in strong alignment across all of the grades. These adjustments are included in the updated version of Prekindergarten through Grade 8 included as Attachment A.

In summary, the Committee has spent hours reviewing and responding to field feedback. The feedback was very helpful and guided our work. Revisions have been made where we felt appropriate. We believe the revised high school performance indicators proposed in this document should be the basis for high school mathematics programs in New York State.

Recommendation 2. The Committee recommends that the performance indicators for the Algebra, Geometry, and Algebra 2 and Trigonometry courses as presented herein be adopted, along with the alignment modifications to Prekindergarten through Grade 8.

C. Regents Examinations for the Three Courses

While the revisions to New York State's mathematics standard and performance indicators were prompted by difficulties with the June 2003 Math A exam, and while the Committee recognizes that tests are not always perfect, the Committee believes that New York State's Regents examinations remain the "gold standard." At the commencement level, the problems inherent with attempting to test students at any other time than after one course year of mathematics became evident with the June 2003 Math A exam, and were documented in the Math A Panel Report. We envision each of the three courses recommended herein as one school year duration for the typical student. It is our belief that, because Regents examinations are the "gold standard" and because external validation is critically important for quality assurance purposes, each of the three courses should culminate in a Regents examination. This would require the development and administration of three Regents examinations in mathematics, not two as is currently the case.

Recommendation 3. The Committee recommends that a Regents examination be developed and administered for each of the three recommended courses.

D. Timing of Implementation

Recommendations for the timing of the implementation were made in the earlier reports; the Committee continues to recommend this implementation timeline:

2005-2006	Prekindergarten through Grade 8
2006-2007	Algebra
2007-2008	Geometry
2008-2009	Algebra 2 and Trigonometry

E. Mathematics Graduation Requirement for a Regents Diploma

Given the current graduation requirement that students must pass the Math A Regents examination (and earn three units of credit of mathematics), and given the Board of Regents decision in October 2003 to approve the change of Math A to a one-year course, the Mathematics Standards Committee crafted the performance indicators for the Algebra course with the understanding that all students will take this course and all students will be expected to pass the Algebra Regents examination to receive a Regents diploma.

As the discussions ensued, and as it became clear that the second course would be a Geometry course, Committee members reflected on the fact that some students who struggle with algebraic symbols and manipulation may very well exhibit a strength in spatial relations. As we considered this, we decided to recommend that the graduation examination requirement in mathematics should be as it is in science, i.e., pass one mathematics Regents examination. Both the Algebra and the Geometry courses are strong courses, and passing a Regents examination in either course would demonstrate a substantial knowledge of mathematics.

We also believe that the current requirement of taking and passing three units of credit of mathematics coursework should remain in place. Because the performance indicators for the Algebra course represent high standards for all students, we recognize that some students might need more than one year to master the content, some perhaps two or even three years. We are recommending that no more than two units of credit be granted for the Algebra course, which will mean that every student will need to earn at least one unit of mathematics credit beyond this course.

For students who take more than one year to pass the Regents examination for the Algebra course, we do not envision that the Geometry course should be the only option available to them. Course options need to be developed locally and should be tailored to the students served. Our thinking is that all students should be exposed to the essential aspects of the Geometry course, such as informal proof and basic geometric relationships. One option we suggest for local consideration is a course that would include some subset of the Geometry course plus some statistics (perhaps using some of the performance indicators from the Algebra 2 and Trigonometry course) and possibly applications to the sciences and social sciences. These issues need to be addressed at the local level.

Recommendation 4. The Committee recommends that the mathematics graduation requirement for a Regents diploma be the passing of one Regents examination in mathematics, and the passing of three units of credit of high school mathematics. The Committee further recommends that the credit granted for the Algebra course be limited to two units.

F. Mathematics Graduation Requirement for a Regents Diploma with Advanced Designation

The Committee believes that, when this is fully implemented, the mathematics requirement for a Regents Diploma with Advanced Designation should be that students must take and pass the three courses recommended herein, and must pass all three Regents examinations. However, with the courses being implemented over a multi-year period, in order to give the system and the students' time to adjust to the new courses, we recommend that the testing requirement for the first two cohorts be that they pass two mathematics Regents examinations.

Recommendation 5. The Committee recommends that the mathematics graduation requirement for a Regents Diploma with Advanced Designation be that students take and pass the three courses of high school mathematics identified herein, and:

- that the first two cohorts pass two of the three Regents examinations in mathematics; and
- that subsequent cohorts pass all three Regents examinations in mathematics.[2]

G. Encouraging More High School Students to Study More Mathematics

Any statewide plan must fit the needs of all children. While some students will struggle to reach the graduation requirements recommended herein for the Regents Diploma, others are already taking four and even five years of mathematics (with many students taking high school mathematics in 8th grade or even earlier). This Committee believes there should be a system in place to recognize and reward students who substantially exceed the high standards required of all.

One possibility we considered was some type of "Math Sequence" diploma designation, which could be awarded to students who, for example, take four years of mathematics and pass all three mathematics Regents examinations. We recognize that such a thought could easily be extended to other subject areas and that this could become problematic. (Clearly, if four or six "Sequence" designations are developed for different academic areas, this could become unwieldy.) As people who love mathematics and as people who believe students should be encouraged to take as much mathematics as possible, this possible complication would not deter us, but we are not insensitive to the potential for enormous challenges.

Therefore, we propose another thought, which we believe is particularly relevant to the issue of a shortage of students entering mathematics and science. Perhaps there should be established a diploma designation such as "Mathematics-Science Sequence." The criteria for such a designation might include taking and passing four years of

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[2] While this Committee is recommending that a third mathematics Regents examination be developed, we recognize there are challenges, both financial resources and staff time, to create, pre-test, field-test and administer another examination. While we note with interest that, currently, science has four Regents examinations developed by the state and mathematics has only two Regents examinations, we are aware that it might not be feasible to create a third mathematics Regents exam right now. Should that be the case, we would recommend that the two examinations be for the first two courses, i.e., Algebra and Geometry. During such time as there are only two examinations, the examination graduation requirements recommended herein would translate to: pass either exam for a Regents Diploma and pass both for the Regents Diploma with Advanced Designation.

mathematics, four years of science, and three Regents examinations in each of the two subjects.

Finally, we believe the State Education Department and the Board of Regents are in a unique position because of their oversight of both Prekindergarten-12 and Higher Education. If consideration is given to a "Mathematics Sequence" or a "Mathematics-Science Sequence" diploma designation, is there some way this might be tied to higher education? Why not connect such a designation with SUNY? We do not have the information available to us to make a concrete recommendation. One thought might be automatic acceptance to SUNY schools for a student who has a "Mathematics-Science Sequence" diploma designation, and an overall average of 80 or more. Another thought might be a 10% or 20% tuition reduction for students who meet criteria like these. Clearly, there are many more possibilities that might be considered.

In summary, the Committee believes that there must be a two-pronged approach, with one prong establishing high expectations for all students to earn a diploma, and with the other prong providing incentives to encourage students to reach even higher.

Recommendation 6. The Committee recommends that ways of rewarding students for taking and mastering more than the graduation requirements be explored and implemented. The Committee members offer to assist in this effort.

III. SUMMARY

The high school mathematics courses recommended in this addendum, we believe, will establish a set of expectations that are high, and that are clear to all, teachers, students, parents, and any other persons in our nation and around the world who review the document. (We are already aware of interest in these proposals from experts in other states and nations.)

We believe that establishing course titles, which are clear to all who know mathematics, enhances clarity. We believe that grouping the performance indicators into mathematically logical courses each year provides focus. We believe that establishing very specific content performance indicators will bring precision of understanding to the field. We believe that the addition of process performance indicators will communicate to all that this Committee sees mathematics not as memorization of procedures, but as a way of thinking, as a way of approaching situations and as a way to solve real-world problems.

IV. CONCLUSION

The members of the Mathematics Standards Committee wish to state our deep appreciation to the staff members of the State Education Department, from Commissioner Mills who entrusted us with this important work, to all of the staff members who worked tirelessly at all hours and on weekends to support our efforts.

We believe that, if our work of developing these performance indicators is followed by curriculum development and by assessments aligned to the performance indicators, and if our classrooms are staffed with teachers who have the necessary background, and who are given the staff development to help children reach these high expectations, New York State will indeed have a "world class" system of mathematics instruction for all of its children.

We all deeply appreciate the opportunity to have been involved with this important work.

Algebra

In implementing the Algebra process and content performance indicators, it is expected that students will identify and justify mathematical relationships. The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate, explore, discover, conjecture, reasoning, argument, justify, explain, proof,* and *apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

Investigate/Explore - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

Discover - Students will make note of possible patterns and generalizations that result from investigation/exploration.

Conjecture - Students will make an overall statement, thought to be true, about the new discovery.

Reasoning - Students will engage in a process that leads to knowing something to be true or false.

Argument - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

Justify/Explain - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

Proof - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems.

Apply - Students will use a theorem or concept to solve an algebraic or numerical problem.

Problem Solving Strand

Students will build new mathematical knowledge through problem solving.

A.PS.1 Use a variety of problem solving strategies to understand new mathematical content

A.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

Students will solve problems that arise in mathematics and in other contexts.

A.PS.3 Observe and explain patterns to formulate generalizations and conjectures

A.PS.4 Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)

Students will apply and adapt a variety of appropriate strategies to solve problems.

A.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)

A.PS.6 Use a variety of strategies to extend solution methods to other problems

A.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

Students will monitor and reflect on the process of mathematical problem solving.

A.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions

A.PS.9 Interpret solutions within the given constraints of a problem

A.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

Reasoning and Proof Strand

Students will recognize reasoning and proof as fundamental aspects of mathematics.

A.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

Students will make and investigate mathematical conjectures.

A.RP.2 Use mathematical strategies to reach a conclusion and provide supportive arguments for a conjecture

A.RP.3 Recognize when an approximation is more appropriate than an exact answer

Students will develop and evaluate mathematical arguments and proofs.

A.RP.4 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

A.RP.5 Construct logical arguments that verify claims or counterexamples that refute them

A.RP.6 Present correct mathematical arguments in a variety of forms

A.RP.7 Evaluate written arguments for validity

Students will select and use various types of reasoning and methods of proof.

A.RP.8 Support an argument by using a systematic approach to test more than one case

A.RP.9 Devise ways to verify results or use counterexamples to refute incorrect statements

A.RP.10 Extend specific results to more general cases

A.RP.11 Use a Venn diagram to support a logical argument

A.RP.12 Apply inductive reasoning in making and supporting mathematical conjectures

Communication Strand

Students will organize and consolidate their mathematical thinking through communication.

A.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

A.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, Venn diagrams, and other diagrams

Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

A.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form.

A.CM.4 Explain relationships among different representations of a problem

A.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

A.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

Students will analyze and evaluate the mathematical thinking and strategies of others.

A.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

A.CM.8 Reflect on strategies of others in relation to one's own strategy

A.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

Students will use the language of mathematics to express mathematical ideas precisely.

A.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures

A.CM.11 Represent word problems using standard mathematical notation

A.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale

A.CM.13 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

Connections Strand

Students will recognize and use connections among mathematical ideas.

A.CN.1 Understand and make connections among multiple representations of the same mathematical idea

A.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

A.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations

A.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

A.CN.5 Understand how quantitative models connect to various physical models and representations

Students will recognize and apply mathematics in contexts outside of mathematics.

A.CN.6 Recognize and apply mathematics to situations in the outside world

A.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics

A.CN.8 Develop an appreciation for the historical development of mathematics

Representation Strand

Students will create and use representations to organize, record, and communicate mathematical ideas.

A.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, and objects created using technology as representations of mathematical concepts

A.R.2 Recognize, compare, and use an array of representational forms

A.R.3 Use representation as a tool for exploring and understanding mathematical ideas

Students will select, apply, and translate among mathematical representations to solve problems.

A.R.4 Select appropriate representations to solve problem situations

A.R.5 Investigate relationships between different representations and their impact on a given problem

Students will use representations to model and interpret physical, social, and mathematical phenomena.

A.R.6 Use mathematics to show and understand physical phenomena (e.g., find the height of a building if a ladder of a given length forms a given angle of elevation with the ground)

A.R.7 Use mathematics to show and understand social phenomena (e.g., determine profit from student and adult ticket sales)

A.R.8 Use mathematics to show and understand mathematical phenomena (e.g., compare the graphs of the functions represented by the equations $y = x^2$ and $y = -x^2$)

Number Sense and Operations Strand

Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.

Number Theory A.N.1 Identify and apply the properties of real numbers (closure, commutative, associative, distributive, identity, inverse) *Note: Students do not need to identify groups and fields, but students should be engaged in the ideas.*

Students will understand meanings of operations and procedures, and how they relate to one another.

Operations A.N.2 Simplify radical terms (no variable in the radicand)

A.N.3 Perform the four arithmetic operations using like and unlike radical terms and express the result in simplest form

A.N.4 Understand and use scientific notation to compute products and quotients of numbers

A.N.5 Solve algebraic problems arising from situations that involve fractions, decimals, percents (decrease/increase and discount), and proportionality/direct variation

A.N.6 Evaluate expressions involving factorial(s), absolute value(s), and exponential expression(s)

A.N.7 Determine the number of possible events, using counting techniques or the Fundamental Principle of Counting

	A.N.8	Determine the number of possible arrangements (permutations) of a list of items

Algebra Strand

Students will represent and analyze algebraically a wide variety of problem solving situations

Variables and Expressions	A.A.1	Translate a quantitative verbal phrase into an algebraic expression
	A.A.2	Write a verbal expression that matches a given mathematical expression
Equations and Inequalities	A.A.3	Distinguish the difference between an algebraic expression and an algebraic equation
	A.A.4	Translate verbal sentences into mathematical equations or inequalities
	A.A.5	Write algebraic equations or inequalities that represent a situation
	A.A.6	Analyze and solve verbal problems whose solution requires solving a linear equation in one variable or linear inequality in one variable
	A.A.7	Analyze and solve verbal problems whose solution requires solving systems of linear equations in two variables
	A.A.8	Analyze and solve verbal problems that involve quadratic equations
	A.A.9	Analyze and solve verbal problems that involve exponential growth and decay
	A.A.10	Solve systems of two linear equations in two variables algebraically (See A.G.7)
	A.A.11	Solve a system of one linear and one quadratic equation

in two variables, where only factoring is required *Note: The quadratic equation should represent a parabola and the solution(s) should be integers.*

Students will perform algebraic procedures accurately.

Variables and Expressions	A.A.12	Multiply and divide monomial expressions with a common base, using the properties of exponents *Note: Use integral exponents only*
	A.A.13	Add, subtract, and multiply monomials and polynomials
	A.A.14	Divide a polynomial by a monomial or binomial, where the quotient has no remainder
	A.A.15	Find values of a variable for which an algebraic fraction is undefined
	A.A.16	Simplify fractions with polynomials in the numerator and denominator by factoring both and renaming them to lowest terms
	A.A.17	Add or subtract fractional expressions with monomial or like binomial denominators
	A.A.18	Multiply and divide algebraic fractions and express the product or quotient in simplest form
	A.A.19	Identify and factor the difference of two perfect squares
	A.A.20	Factor algebraic expressions completely, including trinomials with a lead coefficient of one (after factoring a GCF)
Equations and Inequalities	A.A.21	Determine whether a given value is a solution to a given linear equation in one variable or linear inequality in one variable
	A.A.22	Solve all types of linear equations in one variable
	A.A.23	Solve literal equations for a given variable
	A.A.24	Solve linear inequalities in one variable

	A.A.25	Solve equations involving fractional expressions *Note: Expressions which result in linear equations in one variable.*
	A.A.26	Solve algebraic proportions in one variable which result in linear or quadratic equations
	A.A.27	Understand and apply the multiplication property of zero to solve quadratic equations with integral coefficients and integral roots
	A.A.28	Understand the difference and connection between roots of a quadratic equation and factors of a quadratic expression

Students will recognize, use, and represent algebraically patterns, relations, and functions.

Patterns, Functions, and Relations	A.A.29	Use set-builder notation and/or interval notation to illustrate the elements of a set, given the elements in roster form
	A.A.30	Find the complement of a subset of a given set, within a given universe
	A.A.31	Find the intersection of sets (no more than three sets) and/or union of sets (no more than three sets)
Coordinate Geometry	A.A.32	Explain slope as a rate of change between dependent and independent variables
	A.A.33	Determine the slope of a line, given the coordinates of two points on the line
	A.A.34	Write the equation of a line, given its slope and the coordinates of a point on the line
	A.A.35	Write the equation of a line, given the coordinates of two points on the line
	A.A.36	Write the equation of a line parallel to the x- or y-axis
	A.A.37	Determine the slope of a line, given its equation in any form

	A.A.38	Determine if two lines are parallel, given their equations in any form
	A.A.39	Determine whether a given point is on a line, given the equation of the line
	A.A.40	Determine whether a given point is in the solution set of a system of linear inequalities
	A.A.41	Determine the vertex and axis of symmetry of a parabola, given its equation (See A.G.10)
Trigonometric Functions	A.A.42	Find the sine, cosine, and tangent ratios of an angle of a right triangle, given the lengths of the sides
	A.A.43	Determine the measure of an angle of a right triangle, given the length of any two sides of the triangle
	A.A.44	Find the measure of a side of a right triangle, given an acute angle and the length of another side
	A.A.45	Determine the measure of a third side of a right triangle using the Pythagorean theorem, given the lengths of any two sides

Geometry Strand

Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.

Shapes	A.G.1	Find the area and/or perimeter of figures composed of polygons and circles or sectors of a circle *Note: Figures may include triangles, rectangles, squares, parallelograms, rhombuses, trapezoids, circles, semi-circles, quarter-circles, and regular polygons (perimeter only).*
	A.G.2	Use formulas to calculate volume and surface area of rectangular solids and cylinders

Students will apply coordinate geometry to analyze problem solving situations.

Coordinate Geometry	A.G.3	Determine when a relation is a function, by examining ordered pairs and inspecting graphs of relations

	A.G.4	Identify and graph linear, quadratic (parabolic), absolute value, and exponential functions
	A.G.5	Investigate and generalize how changing the coefficients of a function affects its graph
	A.G.6	Graph linear inequalities
	A.G.7	Graph and solve systems of linear equations and inequalities with rational coefficients in two variables (See A.A.10)
	A.G.8	Find the roots of a parabolic function graphically *Note: Only quadratic equations with integral solutions*
	A.G.9	Solve systems of linear and quadratic equations graphically *Note: Only use systems of linear and quadratic equations that lead to solutions whose coordinates are integers.*
	A.G.10	Determine the vertex and axis of symmetry of a parabola, given its graph (See A.A.41) *Note: The vertex will have an ordered pair of integers and the axis of symmetry will have an integral value.*

Measurement Strand

Students will determine what can be measured and how, using appropriate methods and formulas.

Units of Measurement	A.M.1	Calculate rates using appropriate units (e.g., rate of a space ship versus the rate of a snail)
	A.M.2	Solve problems involving conversions within measurement systems, given the relationship between the units

Understand that all measurement contains error and be able to determine its significance.

Error and Magnitude	A.M.3	Calculate the relative error in measuring square and cubic units, when there is an error in the linear measure

Statistics and Probability Strand

Students will collect, organize, display, and analyze data.

Organization and Display of Data	A.S.1	Categorize data as qualitative or quantitative
	A.S.2	Determine whether the data to be analyzed is univariate or bivariate
	A.S.3	Determine when collected data or display of data may be biased
	A.S.4	Compare and contrast the appropriateness of different measures of central tendency for a given data set
	A.S.5	Construct a histogram, cumulative frequency histogram, and a box-and-whisker plot, given a set of data
	A.S.6	Understand how the five statistical summary (minimum, maximum, and the three quartiles) is used to construct a box-and-whisker plot
	A.S.7	Create a scatter plot of bivariate data
	A.S.8	Construct manually a reasonable line of best fit for a scatter plot and determine the equation of that line
Analysis of Data	A.S.9	Analyze and interpret a frequency distribution table or histogram, a cumulative frequency distribution table or histogram, or a box-and-whisker plot
	A.S.10	Evaluate published reports and graphs that are based on data by considering: experimental design, appropriateness of the data analysis, and the soundness of the conclusions
	A.S.11	Find the percentile rank of an item in a data set and identify the point values for first, second, and third quartiles
	A.S.12	Identify the relationship between the independent and dependent variables from a scatter plot (positive, negative, or none)

A.S.13 Understand the difference between correlation and causation

A.S.14 Identify variables that might have a correlation but not a causal relationship

Students will make predictions that are based upon data analysis.

Predictions from Data

A.S.15 Identify and describe sources of bias and its effect, drawing conclusions from data

A.S.16 Recognize how linear transformations of one-variable data affect the data's mean, median, mode, and range

A.S.17 Use a reasonable line of best fit to make a prediction involving interpolation or extrapolation

Students will understand and apply concepts of probability.

Probability

A.S.18 Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces

A.S.19 Determine the number of elements in a sample space and the number of favorable events

A.S.20 Calculate the probability of an event and its complement

A.S.21 Determine empirical probabilities based on specific sample data

A.S.22 Determine, based on calculated probability of a set of events, if:
- some or all are equally likely to occur
- one is more likely to occur than another
- whether or not an event is certain to happen or not to happen

A.S.23 Calculate the probability of:
- a series of independent events
- a series of dependent events
- two mutually exclusive events
- two events that are not mutually exclusive

Geometry

In implementing the Geometry process and content performance indicators, it is expected that students will identify and justify geometric relationships, formally and informally. For example, students will begin with a definition of a figure and from that definition students will be expected to develop a list of conjectured properties of the figure and to justify each conjecture informally or with formal proof. Students will also be expected to list the assumptions that are needed in order to justify each conjectured property and present their findings in an organized manner.

The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. The variety of approaches to verification and proof is what gives curriculum developers and teachers the flexibility to adapt strategies to address these performance indicators in a manner that meets the diverse needs of our students. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate, explore, discover*, *conjecture, reasoning*, *argument*, *justify, explain, proof,* and *apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

Investigate/Explore - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

Discover - Students will make note of possible relationships of perpendicularity, parallelism, congruence, and/or similarity after investigation/exploration.

Conjecture - Students will make an overall statement, thought to be true, about the new discovery.

Reasoning - Students will engage in a process that leads to knowing something to be true or false.

Argument - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

Justify/Explain - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

Proof - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems using properties of perpendicularity, parallelism, congruence, and similarity with polygons and circles.

Apply - Students will use a theorem or concept to solve a geometric problem.

Problem Solving Strand

Students will build new mathematical knowledge through problem solving.

G.PS.1 Use a variety of problem solving strategies to understand new mathematical content

Students will solve problems that arise in mathematics and in other contexts.

G.PS.2 Observe and explain patterns to formulate generalizations and conjectures

G.PS.3 Use multiple representations to represent and explain problem situations (e.g., spatial, geometric, verbal, numeric, algebraic, and graphical representations)

Students will apply and adapt a variety of appropriate strategies to solve problems.

G.PS.4 Construct various types of reasoning, arguments, justifications and methods of proof for problems

G.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)

G.PS.6 Use a variety of strategies to extend solution methods to other problems

G.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

Students will monitor and reflect on the process of mathematical problem solving.

G.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions

G.PS.9 Interpret solutions within the given constraints of a problem

G.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

Reasoning and Proof Strand

Students will recognize reasoning and proof as fundamental aspects of mathematics.

G.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

G.RP.2 Recognize and verify, where appropriate, geometric relationships of perpendicularity, parallelism, congruence, and similarity, using algebraic strategies

Students will make and investigate mathematical conjectures.

G.RP.3 Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

Students will develop and evaluate mathematical arguments and proofs.

G.RP.4 Provide correct mathematical arguments in response to other students' conjectures, reasoning, and arguments

G.RP.5 Present correct mathematical arguments in a variety of forms

G.RP.6 Evaluate written arguments for validity

Students will select and use various types of reasoning and methods of proof.

G.RP.7 Construct a proof using a variety of methods (e.g., deductive, analytic, transformational)

G.RP.8 Devise ways to verify results or use counterexamples to refute incorrect statements

G.RP.9 Apply inductive reasoning in making and supporting mathematical conjectures

Communication Strand

Students will organize and consolidate their mathematical thinking through communication.

G.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

G.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, and diagrams

Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

G.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form

G.CM.4 Explain relationships among different representations of a problem

G.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

G.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

Students will analyze and evaluate the mathematical thinking and strategies of others.

G.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

G.CM.8 Reflect on strategies of others in relation to one's own strategy

G.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

Students will use the language of mathematics to express mathematical ideas precisely.

G.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures

G.CM.11 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and geometric diagrams

G.CM.12 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

Connections Strand

Students will recognize and use connections among mathematical ideas.

G.CN.1 Understand and make connections among multiple representations of the same mathematical idea

G.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

G.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations

G.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

G.CN.5 Understand how quantitative models connect to various physical models and representations

Students will recognize and apply mathematics in contexts outside of mathematics.

G.CN.6 Recognize and apply mathematics to situations in the outside world

G.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics

G.CN.8 Develop an appreciation for the historical development of mathematics

Representation Strand

Students will create and use representations to organize, record, and communicate mathematical ideas.

G.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, and objects created using technology as representations of mathematical concepts

G.R.2 Recognize, compare, and use an array of representational forms

G.R.3 Use representation as a tool for exploring and understanding mathematical ideas

Students will select, apply, and translate among mathematical representations to solve problems.

G.R.4 Select appropriate representations to solve problem situations

G.R.5 Investigate relationships between different representations and their impact on a given problem

Students will use representations to model and interpret physical, social, and mathematical phenomena.

G.R.6 Use mathematics to show and understand physical phenomena (e.g., determine the number of gallons of water in a fish tank)

G.R.7 Use mathematics to show and understand social phenomena (e.g., determine if conclusions from another person's argument have a logical foundation)

G.R.8 Use mathematics to show and understand mathematical phenomena (e.g., use investigation, discovery, conjecture, reasoning, arguments, justification and proofs to validate that the two base angles of an isosceles triangle are congruent)

Algebra

Note: The algebraic skills and concepts within the Algebra process and content performance indicators must be maintained and applied as students are asked to investigate, make conjectures, give rationale, and justify or prove geometric concepts.

Geometry

Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.

Geometric Relationships

Note: Two-dimensional geometric relationships are addressed in the Informal and Formal Proofs band.

G.G.1 Know and apply that if a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them

G.G.2 Know and apply that through a given point there passes one and only one plane perpendicular to a given line

G.G.3 Know and apply that through a given point there passes one and only one line perpendicular to a given plane

G.G.4 Know and apply that two lines perpendicular to the same plane are coplanar

G.G.5 Know and apply that two planes are perpendicular to each other if and only if one plane contains a line perpendicular to the second plane

G.G.6 Know and apply that if a line is perpendicular to a plane, then any line perpendicular to the given line at its point of intersection with the given plane is in the given plane

G.G.7 Know and apply that if a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane

G.G.8 Know and apply that if a plane intersects two parallel planes, then the intersection is two parallel lines

G.G.9 Know and apply that if two planes are perpendicular to the same line, they are parallel

G.G.10 Know and apply that the lateral edges of a prism are congruent and parallel

G.G.11 Know and apply that two prisms have equal volumes if their bases have equal areas and their altitudes are equal

G.G.12 Know and apply that the volume of a prism is the product of the area of the base and the altitude

G.G.13 Apply the properties of a regular pyramid, including:

- lateral edges are congruent
- lateral faces are congruent isosceles triangles
- volume of a pyramid equals one-third the product of the area of the base and the altitude

G.G.14 Apply the properties of a cylinder, including:

- bases are congruent
- volume equals the product of the area of the base and the altitude
- lateral area of a right circular cylinder equals the product of an altitude and the circumference of the base

G.G.15 Apply the properties of a right circular cone, including:

- lateral area equals one-half the product of the slant height and the circumference of its base

- o volume is one-third the product of the area of its base and its altitude

G.G.16 Apply the properties of a sphere, including:
- o the intersection of a plane and a sphere is a circle
- o a great circle is the largest circle that can be drawn on a sphere
- o two planes equidistant from the center of the sphere and intersecting the sphere do so in congruent circles
- o surface area is $4\pi r^2$
- o volume is $\frac{4}{3}\pi r^3$

Constructions

G.G.17 Construct a bisector of a given angle, using a straightedge and compass, and justify the construction

G.G.18 Construct the perpendicular bisector of a given segment, using a straightedge and compass, and justify the construction

G.G.19 Construct lines parallel (or perpendicular) to a given line through a given point, using a straightedge and compass, and justify the construction

G.G.20 Construct an equilateral triangle, using a straightedge and compass, and justify the construction

Locus

G.G.21 Investigate and apply the concurrence of medians, altitudes, angle bisectors, and perpendicular bisectors of triangles

G.G.22 Solve problems using compound loci

G.G.23 Graph and solve compound loci in the coordinate plane

Students will identify and justify geometric relationships formally and informally.

Informal and Formal Proofs

G.G.24 Determine the negation of a statement and establish its truth value

G.G.25 Know and apply the conditions under which a compound statement (conjunction, disjunction, conditional, biconditional) is true

G.G.26 Identify and write the inverse, converse, and contrapositive of a given conditional statement and note the logical equivalences

G.G.27 Write a proof arguing from a given hypothesis to a given conclusion

G.G.28 Determine the congruence of two triangles by using one of the five congruence techniques (SSS, SAS, ASA, AAS, HL), given sufficient information about the sides and/or angles of two congruent triangles

G.G.29 Identify corresponding parts of congruent triangles

G.G.30 Investigate, justify, and apply theorems about the sum of the measures of the angles of a triangle

G.G.31 Investigate, justify, and apply the isosceles triangle theorem and its converse

G.G.32 Investigate, justify, and apply theorems about geometric inequalities, using the exterior angle theorem

G.G.33 Investigate, justify, and apply the triangle inequality theorem

G.G.34 Determine either the longest side of a triangle given the three angle measures or the largest angle given the lengths of three sides of a triangle

G.G.35 Determine if two lines cut by a transversal are parallel, based on the measure of given pairs of angles formed by the transversal and the lines

G.G.36 Investigate, justify, and apply theorems about the sum of the measures of the interior and exterior angles of polygons

G.G.37 Investigate, justify, and apply theorems about each interior and exterior angle measure of regular polygons

G.G.38 Investigate, justify, and apply theorems about parallelograms involving their angles, sides, and diagonals

G.G.39 Investigate, justify, and apply theorems about special parallelograms (rectangles, rhombuses, squares) involving their angles, sides, and diagonals

G.G.40 Investigate, justify, and apply theorems about trapezoids (including isosceles trapezoids) involving their angles, sides, medians, and diagonals

G.G.41 Justify that some quadrilaterals are parallelograms, rhombuses, rectangles, squares, or trapezoids

G.G.42 Investigate, justify, and apply theorems about geometric relationships, based on the properties of the line segment joining the midpoints of two sides of the triangle

G.G.43 Investigate, justify, and apply theorems about the centroid of a triangle, dividing each median into segments whose lengths are in the ratio 2:1

G.G.44 Establish similarity of triangles, using the following theorems: AA, SAS, and SSS

G.G.45 Investigate, justify, and apply theorems about similar triangles

G.G.46 Investigate, justify, and apply theorems about proportional relationships among the segments of the sides of the triangle, given one or more lines parallel to one side of a triangle and intersecting the other two sides of the triangle

G.G.47 Investigate, justify, and apply theorems about mean proportionality:

- the altitude to the hypotenuse of a right triangle is the mean proportional between the two segments along the hypotenuse
- the altitude to the hypotenuse of a right triangle divides the hypotenuse so that either leg of the right triangle is the mean proportional between the hypotenuse and segment of the hypotenuse adjacent to that leg

G.G.48 Investigate, justify, and apply the Pythagorean theorem and its converse

G.G.49 Investigate, justify, and apply theorems regarding chords of a circle:
- perpendicular bisectors of chords
- the relative lengths of chords as compared to their distance from the center of the circle

G.G.50 Investigate, justify, and apply theorems about tangent lines to a circle:
- a perpendicular to the tangent at the point of tangency
- two tangents to a circle from the same external point
- common tangents of two non-intersecting or tangent circles

G.G.51 Investigate, justify, and apply theorems about the arcs determined by the rays of angles formed by two lines intersecting a circle when the vertex is:
- inside the circle (two chords)
- on the circle (tangent and chord)
- outside the circle (two tangents, two secants, or tangent and secant)

G.G.52 Investigate, justify, and apply theorems about arcs of a circle cut by two parallel lines

G.G.53 Investigate, justify, and apply theorems regarding segments intersected by a circle:
- along two tangents from the same external point
- along two secants from the same external point
- along a tangent and a secant from the same external point
- along two intersecting chords of a given circle

Students will apply transformations and symmetry to analyze problem solving situations.

Transformational Geometry

G.G.54 Define, investigate, justify, and apply isometries in the plane (rotations, reflections, translations, glide reflections) *Note: Use proper function notation.*

G.G.55 Investigate, justify, and apply the properties that remain invariant under translations, rotations, reflections, and glide reflections

G.G.56 Identify specific isometries by observing orientation, numbers of invariant points, and/or parallelism

G.G.57 Justify geometric relationships (perpendicularity, parallelism, congruence) using transformational techniques (translations, rotations, reflections)

G.G.58 Define, investigate, justify, and apply similarities (dilations and the composition of dilations and isometries)

G.G.59 Investigate, justify, and apply the properties that remain invariant under similarities

G.G.60 Identify specific similarities by observing orientation, numbers of invariant points, and/or parallelism

G.G.61 Investigate, justify, and apply the analytical representations for translations, rotations about the origin of 90º and 180º, reflections over the lines $x = 0$, $y = 0$, and $y = x$, and dilations centered at the origin

Students will apply coordinate geometry to analyze problem solving situations

Coordinate Geometry

G.G.62 Find the slope of a perpendicular line, given the equation of a line

G.G.63 Determine whether two lines are parallel, perpendicular, or neither, given their equations

G.G.64 Find the equation of a line, given a point on the line and the equation of a line perpendicular to the given line

G.G.65 Find the equation of a line, given a point on the line and the equation of a line parallel to the desired line

G.G.66 Find the midpoint of a line segment, given its endpoints

G.G.67 Find the length of a line segment, given its endpoints

G.G.68 Find the equation of a line that is the perpendicular bisector of a line segment, given the endpoints of the line segment

G.G.69 Investigate, justify, and apply the properties of triangles and quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas

G.G.70 Solve systems of equations involving one linear equation and one quadratic equation graphically

G.G.71 Write the equation of a circle, given its center and radius or given the endpoints of a diameter

G.G.72 Write the equation of a circle, given its graph
Note: The center is an ordered pair of integers and the radius is an integer.

G.G.73 Find the center and radius of a circle, given the equation of the circle in center-radius form

G.G.74 Graph circles of the form $(x-h)^2 + (j-k)^2 = r^2$

Algebra 2 and Trigonometry

In implementing the Algebra 2 and Trigonometry process and content performance indicators, it is expected that students will identify and justify mathematical relationships, formally and informally. The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate, explore, discover, conjecture, reasoning, argument, justify, explain, proof,* and *apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

Investigate/Explore - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

Discover - Students will make note of possible patterns and generalizations that result from investigation/exploration.

Conjecture - Students will make an overall statement, thought to be true, about the new discovery.

Reasoning - Students will engage in a process that leads to knowing something to be true or false.

Argument - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

Justify/Explain - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

Proof - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems.

Apply - Students will use a theorem or concept to solve an algebraic or numerical problem.

Problem Solving Strand

Students will build new mathematical knowledge through problem solving.

A2.PS.1 Use a variety of problem solving strategies to understand new mathematical content

A2.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

Students will solve problems that arise in mathematics and in other contexts.

A2.PS.3 Observe and explain patterns to formulate generalizations and conjectures

A2.PS.4 Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)

Students will apply and adapt a variety of appropriate strategies to solve problems.

A2.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)

A2.PS.6 Use a variety of strategies to extend solution methods to other problems

A2.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

Students will monitor and reflect on the process of mathematical problem solving.

A2.PS.8 Determine information required to solve the problem, choose methods for obtaining the information, and define parameters for acceptable solutions

A2.PS.9 Interpret solutions within the given constraints of a problem

A2.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

Reasoning and Proof Strand

Students will recognize reasoning and proof as fundamental aspects of mathematics.

A2.RP.1 Support mathematical ideas using a variety of strategies

Students will make and investigate mathematical conjectures.

A2.RP.2 Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

A2.RP.3 Evaluate conjectures and recognize when an estimate or approximation is more appropriate than an exact answer

A2.RP.4 Recognize when an approximation is more appropriate than an exact answer

Students will develop and evaluate mathematical arguments and proofs.

A2.RP.5 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

A2.RP.6 Construct logical arguments that verify claims or counterexamples that refute claims

A2.RP.7 Present correct mathematical arguments in a variety of forms

A2.RP.8 Evaluate written arguments for validity

Students will select and use various types of reasoning and methods of proof.

A2.RP.9 Support an argument by using a systematic approach to test more than one case

A2.RP.10 Devise ways to verify results, using counterexamples and informal indirect proof

A2.RP.11 Extend specific results to more general cases

A2.RP.12 Apply inductive reasoning in making and supporting mathematical conjectures

Communication Strand

Students will organize and consolidate their mathematical thinking through communication.

A2.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

A2.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, and diagrams

Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

A2.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form

A2.CM.4 Explain relationships among different representations of a problem

A2.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

A2.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

Students will analyze and evaluate the mathematical thinking and strategies of others.

A2.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

A2.CM.8 Reflect on strategies of others in relation to one's own strategy

A2.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

Students will use the language of mathematics to express mathematical ideas precisely.

A2.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures

A2.CM.11 Represent word problems using standard mathematical notation

A2.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale

A2.CM.13 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

Connections Strand

Students will recognize and use connections among mathematical ideas.

A2.CN.1 Understand and make connections among multiple representations of the same mathematical idea

A2.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

A2.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations

A2.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

A2.CN.5 Understand how quantitative models connect to various physical models and representations

Students will recognize and apply mathematics in contexts outside of mathematics.

A2.CN.6 Recognize and apply mathematics to situations in the outside world

A2.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics

A2.CN.8 Develop an appreciation for the historical development of mathematics

Representation Strand

Students will create and use representations to organize, record, and communicate mathematical ideas.

A2.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, and objects created using technology as representations of mathematical concepts

A2.R.2 Recognize, compare, and use an array of representational forms

A2.R.3 Use representation as a tool for exploring and understanding mathematical ideas

Students will select, apply, and translate among mathematical representations to solve problems.

A2.R.4 Select appropriate representations to solve problem situations

A2.R.5 Investigate relationships among different representations and their impact on a given problem

Students will use representations to model and interpret physical, social, and mathematical phenomena.

A2.R.6 Use mathematics to show and understand physical phenomena (e.g., investigate sound waves using the sine and cosine functions)

A2.R.7 Use mathematics to show and understand social phenomena (e.g., interpret the results of an opinion poll)

A2.R.8 Use mathematics to show and understand mathematical phenomena (e.g., use random number generator to simulate a coin toss)

Number Sense and Operations Strand

Students will understand meanings of operations and procedures, and how they relate to one another.

Operations A2.N.1 Evaluate numerical expressions with negative and/or fractional exponents, without the aid of a calculator (when the answers are rational numbers)

	A2.N.2	Perform arithmetic operations (addition, subtraction, multiplication, division) with expressions containing irrational numbers in radical form
	A2.N.3	Perform arithmetic operations with polynomial expressions containing rational coefficients
	A2.N.4	Perform arithmetic operations on irrational expressions
	A2.N.4	Rationalize a denominator containing a radical expression
	A2.N.5	Write square roots of negative numbers in terms of *i*
	A2.N.6	Simplify powers of *i*
	A2.N.7	Determine the conjugate of a complex number
	A2.N.8	Perform arithmetic operations on complex numbers and write the answer in the form $a+bi$. *Note: This includes simplifying expressions with complex denominators.*
	A2.N.9	Know and apply sigma notation

Algebra Strand

Students will represent and analyze algebraically a wide variety of problem solving situations.

Equations and Inequalities	A2.A.1	Solve absolute value equations and inequalities involving linear expressions in one variable
	A2.A.2	Use the discriminant to determine the nature of the roots of a quadratic equation
	A2.A.3	Solve systems of equations involving one linear equation and one quadratic equation algebraically *Note: This includes rational equations that result in linear equations with extraneous roots.*
	A2.A.4	Solve quadratic inequalities in one and two variables, algebraically and graphically

	A2.A.5	Use direct and inverse variation to solve for unknown values
	A2.A.6	Solve an application which results in an exponential function

Students will perform algebraic procedures accurately.

Variables and Expressions	A2.A.7	Factor polynomial expressions completely, using any combination of the following techniques: common factor extraction, difference of two perfect squares, quadratic trinomials
	A2.A.8	Apply the rules of exponents to simplify expressions involving negative and/or fractional exponents
	A2.A.9	Rewrite algebraic expressions that contain negative exponents using only positive exponents
	A2.A.10	Rewrite algebraic expressions with fractional exponents as radical expressions
	A2.A.11	Rewrite algebraic expressions in radical form as expressions with fractional exponents
	A2.A.12	Evaluate exponential expressions, including those with base e
	A2.A.13	Simplify radical expressions
	A2.A.14	Perform addition, subtraction, multiplication and division of radical expressions
	A2.A.15	Rationalize denominators involving algebraic radical expressions
	A2.A.16	Perform arithmetic operations with rational expressions and rename to lowest terms
	A2.A.17	Simplify complex fractional expressions
	A2.A.18	Evaluate logarithmic expressions in any base
	A2.A.19	Apply the properties of logarithms to rewrite logarithmic expressions in equivalent forms

Equations and Inequalities	A2.A.20	Determine the sum and product of the roots of a quadratic equation by examining its coefficients
	A2.A.21	Determine the quadratic equation, given the sum and product of its roots
	A2.A.22	Solve radical equations
	A2.A.23	Solve rational equations and inequalities
	A2.A.24	Know and apply the technique of completing the square
	A2.A.25	Solve quadratic equations, using the quadratic formula
	A2.A.26	Find the solution to polynomial equations of higher degree that can be solved using factoring and/or the quadratic formula
	A2.A.27	Solve exponential equations with and without common bases
	A2.A.28	Solve a logarithmic equation by rewriting as an exponential equation

Students will recognize, use, and represent algebraically patterns, relations, and functions.

Patterns, Functions, and Relations	A2.A.29	Identify an arithmetic or geometric sequence and find the formula for its *n*th term
	A2.A.30	Determine the common difference in an arithmetic sequence
	A2.A.31	Determine the common ratio in a geometric sequence
	A2.A.32	Determine a specified term of an arithmetic or geometric sequence
	A2.A.33	Specify terms of a sequence, given its recursive definition
	A2.A.34	Represent the sum of a series, using sigma notation
	A2.A.35	Determine the sum of the first *n* terms of an arithmetic or geometric series

	A2.A.36	Apply the binomial theorem to expand a binomial and determine a specific term of a binomial expansion
	A2.A.37	Define a relation and function
	A2.A.38	Determine when a relation is a function
	A2.A.39	Determine the domain and range of a function from its equation
	A2.A.40	Write functions in functional notation
	A2.A.41	Use functional notation to evaluate functions for given values in the domain
	A2.A.42	Find the composition of functions
	A2.A.43	Determine if a function is one-to-one, onto, or both
	A2.A.44	Define the inverse of a function
	A2.A.45	Determine the inverse of a function and use composition to justify the result
	A2.A.46	Perform transformations with functions and relations: $f(x+a)$, $f(x)+a$, $f(-x)$, $-f(x)$, $af(x)$
Coordinate Geometry	A2.A.47	Determine the center-radius form for the equation of a circle in standard form
	A2.A.48	Write the equation of a circle, given its center and a point on the circle
	A2.A.49	Write the equation of a circle from its graph
	A2.A.50	Approximate the solution to polynomial equations of higher degree by inspecting the graph
	A2.A.51	Determine the domain and range of a function from its graph
	A2.A.52	Identify relations and functions, using graphs

	A2.A.53	Graph exponential functions of the form $y = b^x$ for positive values of *b*, including $b = e$
	A2.A.54	Graph logarithmic functions, using the inverse of the related exponential function
Trigonometric Functions	A2.A.55	Express and apply the six trigonometric functions as ratios of the sides of a right triangle
	A2.A.56	Know the exact and approximate values of the sine, cosine, and tangent of 0º, 30º, 45º, 60º, 90º, 180º, and 270º angles
	A2.A.57	Sketch and use the reference angle for angles in standard position
	A2.A.58	Know and apply the co-function and reciprocal relationships between trigonometric ratios
	A2.A.59	Use the reciprocal and co-function relationships to find the value of the secant, cosecant, and cotangent of 0º, 30º, 45º, 60º, 90º, 180º, and 270º angles
	A2.A.60	Sketch the unit circle and represent angles in standard position
	A2.A.61	Determine the length of an arc of a circle, given its radius and the measure of its central angle
	A2.A.62	Find the value of trigonometric functions, if given a point on the terminal side of angle θ
	A2.A.63	Restrict the domain of the sine, cosine, and tangent functions to ensure the existence of an inverse function
	A2.A.64	Use inverse functions to find the measure of an angle, given its sine, cosine, or tangent
	A2.A.65	Sketch the graph of the inverses of the sine, cosine, and tangent functions
	A2.A.66	Determine the trigonometric functions of any angle, using technology
	A2.A.67	Justify the Pythagorean identities

A2.A.68 Solve trigonometric equations for all values of the variable from 0º to 360º

A2.A.69 Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function

A2.A.70 Sketch and recognize one cycle of a function of the form $y = A \sin Bx$ or $y = A \cos Bx$

A2.A.71 Sketch and recognize the graphs of the functions $y = \sec(x)$, $y = \csc(x)$, $y = \tan(x)$, and $y = \cot(x)$

A2.A.72 Write the trigonometric function that is represented by a given periodic graph

A2.A.73 Solve for an unknown side or angle, using the Law of Sines or the Law of Cosines

A2.A.74 Determine the area of a triangle or a parallelogram, given the measure of two sides and the included angle

A2.A.75 Determine the solution(s) from the SSA situation (ambiguous case)

A2.A.76 Apply the angle sum and difference formulas for trigonometric functions

A2.A.77 Apply the double-angle and half-angle formulas for trigonometric functions

Measurement Strand

Students will determine what can be measured and how, using appropriate methods and formulas.

Units of Measurement

A2.M.1 Define radian measure

A2.M.2 Convert between radian and degree measures

Statistics and Probability Strand

Students will collect, organize, display, and analyze data.

Collection of Data	A2.S.1	Understand the differences among various kinds of studies (e.g., survey, observation, controlled experiment)
	A2.S.2	Determine factors which may affect the outcome of a survey
Organization and Display of Data	A2.S.3	Calculate measures of central tendency with group frequency distributions
	A2.S.4	Calculate measures of dispersion (range, quartiles, interquartile range, standard deviation, variance) for both samples and populations
	A2.S.5	Know and apply the characteristics of the normal distribution

Students will make predictions that are based upon data analysis.

Predictions from Data	A2.S.6	Determine from a scatter plot whether a linear, logarithmic, exponential, or power regression model is most appropriate
	A2.S.7	Determine the function for the regression model, using appropriate technology, and use the regression function to interpolate and extrapolate from the data
	A2.S.8	Interpret within the linear regression model the value of the correlation coefficient as a measure of the strength of the relationship

Students will understand and apply concepts of probability.

Probability	A2.S.9	Differentiate between situations requiring permutations and those requiring combinations
	A2.S.10	Calculate the number of possible permutations $({}_nP_r)$ of n items taken r at a time
	A2.S.11	Calculate the number of possible combinations $({}_nC_r)$ of n items taken r at a time

A2.S.12 Use permutations, combinations, and the Fundamental Principle of Counting to determine the number of elements in a sample space and a specific subset (event)

A2.S.13 Calculate theoretical probabilities, including geometric applications

A2.S.14 Calculate empirical probabilities

A2.S.15 Know and apply the binomial probability formula to events involving the terms *exactly*, *at least*, and *at most*

A2.S.16 Use the normal distribution as an approximation for binomial probabilities